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APPLICATION OF THE BARON-MYERSON MONOPOLIST REGULATION MECHANISM: ISSUES ON SELECTING THE COST PROBABILITY DISTRIBUTIONS

by

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June 1994

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The end of cold war levels of defense expenditures has promoted the reduction in the number of defense-related companies, creating potential monopolistic economic scenarios for defense procurement. This thesis studies one methodology to deal with these scenarios, based on the Baron-Myerson monopolist regulation mechanisms.

The Baron-Myerson mechanism provides a tool to regulate monopolists when their costs are unknown or cannot be measured, because it is designed as to compel the producer to reveal its costs by maximizing the company's profit when it announces their true value. The government presents a modified purchasing plan to the producer, buys according to the announced costs and pays a subsidy (or levies a tax) to the producer.

To apply Baron-Myerson the government needs to know the demand for the good or service it requires, and an estimate of probability density function for the possible costs of the project. This second assumption is the issue addressed in this thesis.

The thesis establishes selection criteria and policy recommendations that the government can use to choose a probability density function for the application of Baron-Myerson. The criteria is based on the maximization of the expected government gain, given the level of efficiency of the producer. Also, an analysis of the policy implications of the government's choice is made, to determine the effects of a change in policy on the total welfare, the firm's profits and the government gain.

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Application of the Baron-Myerson Monopolist Regulation Mechanism: Issues on Selecting the Cost Probability Distributions

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ABSTRACT

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I. INTRODUCTION.

As the defense industrial base becomes smaller due to the reduced demand of defense goods, the problem of dealing with a small number of contractors to procure a good or service becomes significant for the government. The end of cold war levels of defense expenditures has promoted the consolidation of companies and a reduction in the number of service providers for the Department of Defense, creating monopolistic economic scenarios for defense procurement. This thesis studies one methodology to deal with these scenarios, based on the Loeb-Magat and Baron-Myerson monopolist regulation mechanisms.

These mechanism were originally motivated by the need to regulate public utilities such as electric and telephone service providers, and must be adapted to deal with the defense environment. In a common regulation scheme, a government agency imposes certain rules upon the providers of goods and services to avoid the excessive extraction of surplus from the consumers by the producers. For the purposes of this thesis, the regulator and the consumers are the same: the defense agency that needs to procure a good or service. This condition simplifies many of the assumptions that the regulation mechanism requires, such as knowing the demand the government has for goods and services provided by a contractor.

The privileged information that the contractors possesses about the technology and effort needed to produce the goods and services, gives them an inherent informational advantage over the government. The government has to "guess" the range of these parameters and base its policy on this belief, since the real values are only known to the contractors. This thesis studies the effect of this "educated guess" in the outcome of the application of the Baron-Myerson mechanism to defense procurement. The outcome is measured in terms of expected gain (either of the government or of the producer of the good or service) since only a statistical description is available for the cost parameters.

To focus the study of the effect of the government belief about the contractor's costs, this thesis considers only one dimension of the problem: the relative distribution of the cost parameters. The government belief will be "more favorable" as it consider firms with costs parameters that take relatively low values ("efficient" firms). Similarly, the government belief will be "less favorable" as it consider firms with cost parameters that take relatively high values ("inefficient" firms). Therefore, this thesis would help to predict the consequences of pursuing different policies regarding the procurement of goods in the overall welfare of the consumer (government) and the producer (firm). The policies may be either aggressive, when they allow only the efficient contractors to operate, or conservative, allowing less efficient firms contribute to the outcome.

Summarizing, the major goal of this work is to provide policy guidelines for the selection of a "government belief", based on its impact on the expected outcome that this policy will produce. Previous theses have analyzed the problem using uniform and linear probability density functions. In this thesis, the analysis is extended to a larger class of probability density functions.

In Chapter II, the basic theoretical tools needed for the analysis of the problem are described, mostly in a qualitative manner to help the intuitive understanding of the methods involved in the policy. Also, a more precise definition of what a "more favorable" decision means is outlined.

Chapter III summarizes the main results obtained from the analysis, giving the conditions under which the government gain is maximized. The basic assumption made in this chapter is that the government does not know the real cost distribution of the contractor, therefore accepting a mismatch between the reality and the government's belief.

The effect on the contractor's behavior is also studied, since the application of the Baron-Myerson mechanism will create very specific incentive structures for the firm.

In Chapter IV the strategic choice problem of the government is analyzed. This chapter provides guidelines for the selection of a policy instrument when the government is faced with more than one option to apply the Baron-Myerson mechanism.

Finally, Chapter V summarizes the conclusions of the study and recommendations are formulated for the application of this methodology in the procurement arena.

II. THEORETICAL BACKGROUND.

This chapter introduces the basic theoretical results and practical applications of some of the previous efforts for the regulation of monopolists that have an informational advantage over the institutions assigned to govern them. First, the notion of truth revealing mechanism is introduced by the explanation of the Loeb-Magat regulation mechanism [Ref. 1]. Next, this basic approach is extended as Baron and Myerson did [Ref. 2], to improve the outcome for the consumers and regulator body. In this chapter most of the explanations will be qualitative so that the principles behind the methods are clearly demonstrated, instead of burying the notions under quantitative results.

As noted in the introduction, for the purposes of this thesis the regulator and the government are the same entity. It is usual in the defense procurement arena that the only consumer is the government and that the firms that provide goods and services enjoy a certain degree of monopoly power because of being the sole provider of a specific technology.

A. THE LOEB-MAGAT MODEL FOR MONOPOLIST REGULATION.

The Loeb-Magat mechanism for controlling a monopolist is based on the idea of a tradeoff between an accurate estimate of the regulated industry's costs and government surplus (or gain). In its basic form, the government gives up all its surplus in exchange for the information about the marginal costs of the producer, to guarantee the allocative efficiency of the outcome. The regulator fixes the price so it is equal to the "revealed" marginal cost, in this way achieving the competitive outcome where the price paid by the government equals to the marginal cost of providing the service or producing the good.

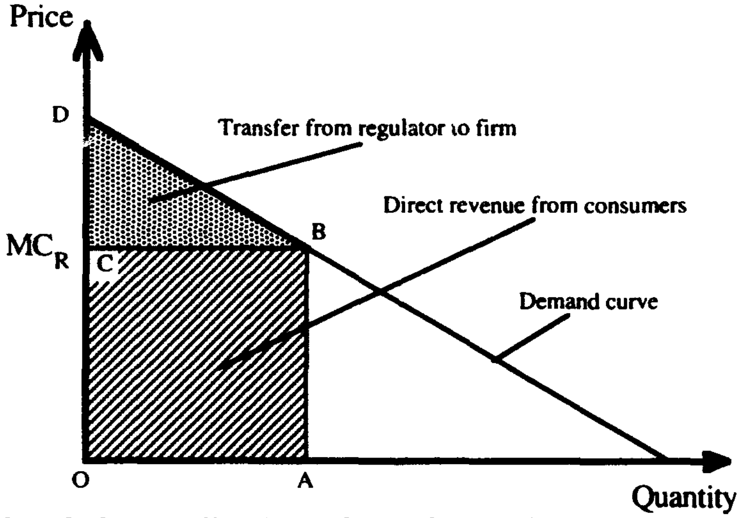
To see the truth revealing nature of this mechanism, consider the situation shown in Figure 1. The demand curve for the good or service is assumed known by both the government and the producer of the good or service. The shaded area CBD represents the profit (net revenue) that the company earns if it marginal cost is MC_R, assumed to be the true marginal cost (Total cost is area OABC, assuming that the marginal cost is constant and there are no fixed costs). Consider now the situation where the company decides to report a different cost, $MC_{R'}$. Figure 2 shows the situation when $MC_{R'}>MC_R$. In this case, the company earns a profit equal to the area HGBD, that is smaller than the amount received if it reports MC_R (Area HED). The solid triangle BEG represents the lost profit due to the misrepresentation of the cost. Then, the firm has no incentive to overrepresent costs, because it losses profit when it does.

Figure 3 summarizes both the cost overrepresentation and underrepresentation situations. Triangle EGB is the lost profit due to underreporting the costs. Triangle LEJ is the lost profit due to overreporting the costs. Therefore, it is in the own interest of the company, if its goal is to maximize profits, to report the real marginal cost to the regulator.

The main problem with this approach is that the government gives away all its surplus, although it receives the product or service at a competitive price level. A partial solution to this problem is to charge the producer a fixed tax, high enough to recover most of the surplus, but low enough to keep the mechanism useful. If the tax is too high, the company may not cover its fixed costs, and the incentive structure designed to reveal the producer's marginal cost breaks down. The additional information needed by the government is an upper bound for the producer's marginal cost (UBMC). Since this information is kept private by the firm, the government must rely on its belief about the UBMC to formulate the purchasing policy.

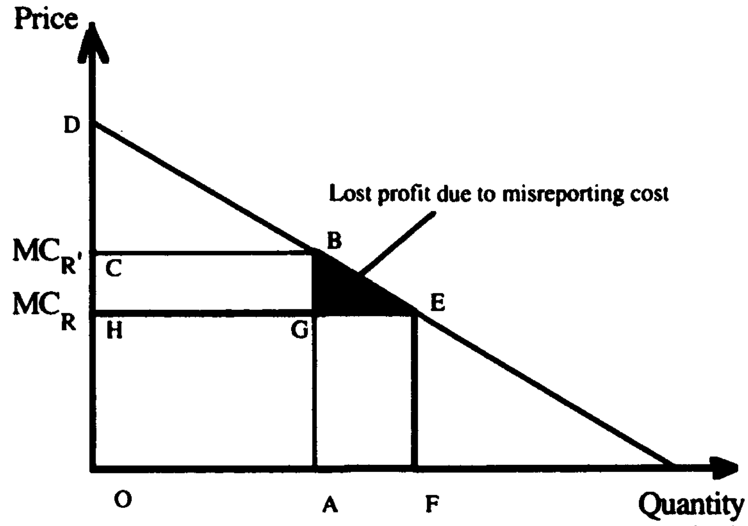
Summarizing, the Loeb-Magat mechanism has the interesting property of compelling the revelation of the producer's true marginal cost, assuming that the government and the producer share the knowledge of the demand curve for the good or service, and that the producer is able to determine its own marginal cost. Under the additional assumption that the regulator is able to estimate an upper bound for the marginal cost, it is possible to recover most of the surplus given away by the application of a lumped-sum tax based on this government's belief.

FIGURE 1



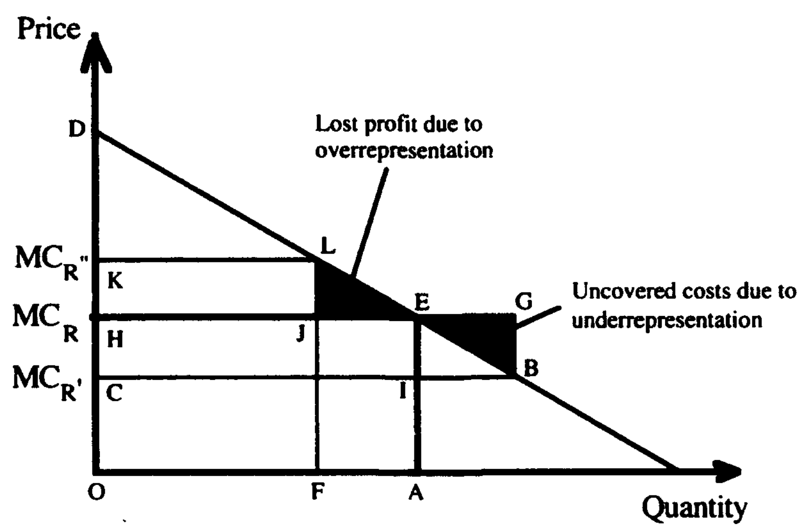
Example of the application of the Loeb-Magat regulation model.

FIGURE 2



Example of the application of the Loeb-Magat regulation model: Reported marginal cost higher than real marginal cost.

FIGURE 3



Example of the application of the Loeb-Magat regulation model: summary of the truth revealing mechanism operation.

B. THE BARON-MYERSON MODEL FOR MONOPOLIST REGULATION.

Baron and Myerson extended the Loeb-Magat model by requiring additional assumptions about the unknown cost parameters of the regulated firm, and at the same time offering a mechanism that enables the government to extract more rent from the firm. Both methods share the same basic principle of truth revelation. The firm maximizes its profits only if it reports its true cost parameter. The government uses the same Loeb-Magat model, though to extract some rent from the firm, it does not use the real demand curve but a modified purchasing plan derived from the demand curve and the statistical properties of the unknown cost parameter[1].

The Baron-Myerson (B-M) mechanism explicitly incorporates a transfer payment from the government to the firm, although it may sometimes be negative (i.e., a tax). Also, the B-M mechanism is posed as an optimization problem, where the firm's profit is given a variable weight. The goal is to maximize the weighted sum of the expected government's surplus and expected firm's profit. Notably, the method has a meaningful result even if the expected firm's profits are given no weight in the optimization problem. If both expected surpluses are given the same weight, the Loeb-Magat result is obtained. For the defense procurement case, the most interesting case occurs when no weight is given to the expected firm's profit, since the government would maximize its own expected gain.

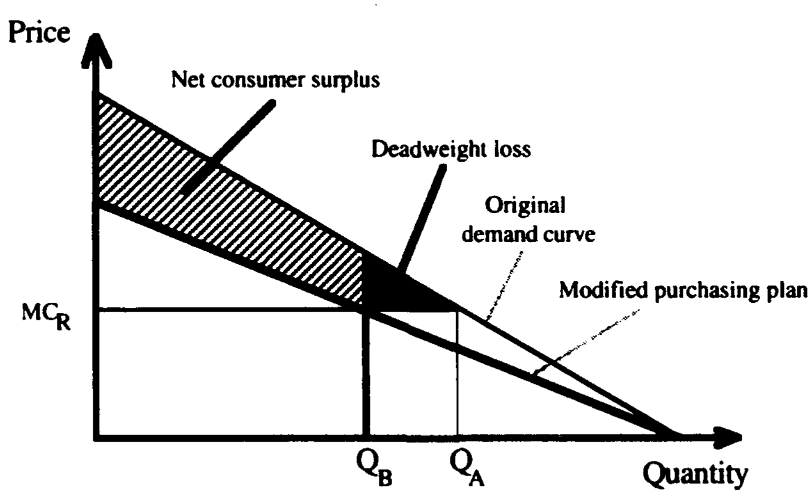
The additional assumption required by B-M is the availability of an estimate of a probability density function (or equivalently the cumulative distribution function) for the cost parameter, instead of just the upper bound. This assumption is more challenging, because the regulator will need much more information compared with the Loeb-Magat case to formulate its belief about the unknown cost parameter.

Figure 4 shows an example of the application of the B-M mechanism. For this example, the government does not know the marginal cost of the firm, and the fixed costs

[1]This interpretation of the Baron-Myerson mechanism comes from [Ref. 3]

incurred are assumed to be zero. Also, the figure sets the origin of the price axis at the lower bound for the marginal cost, and assumes that the upper bound of the marginal cost is higher than the highest price that the government is willing to pay for the product or service. These simplifications may be done without a loss of generality. The government believes that the firm has a marginal cost that is distributed uniformly. The modified purchasing plan depends on the original demand curve, the probability distribution function for the unknown marginal cost and the weight given to the firm's profits in the overall optimization problem. The gain for the government increases as the weight given to the profit decreases, and is bigger than in the Loeb-Magat case. Since the B-M is conceptually the same than the Loeb-Magat case, except for the use of a modified purchasing plan, the truth revealing principle may be explained in a similar way. There is a deadweight loss associated with the application of the method, due to a decrease in consumption. If the firm reported its real costs without the forcing mechanism, the consumers would acquire Q_A. Instead, they buy only Q_B.[2]

FIGURE 4



Example of the application of the B-M regulation model: recovering some of the consumer's surplus using a modified purchasing plan.

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[2]For other properties of the B-M mechanism, see [Ref. 2]

Summarizing, the B-M model is an extension to the Loeb-Magat model, requiring estimates for: (i) the demand curve for the good or service; and (ii) a probability density function for the unknown cost parameter. Using this additional information, the government is able to extract rent from the firm and avoid giving up most of its surplus. In the following section, a specific criteria will be defined to qualify a cost distribution function when compared to another.

C. CHARACTERIZATION OF THE PROBABILITY DISTRIBUTION FUNCTION USED IN THE B-M MECHANISM.

Since one of the main topics of this thesis is to determine under which circumstances the government is better off by choosing one cost parameter distribution over another, it is necessary to clearly define the criteria used for the comparison of the distribution functions. In this section one criterion is defined, though it is not necessarily the only criteria that may be used to compare cost distribution functions.

Consider an unknown cost parameter Θ, that may take any value in the interval $[\theta_0, \theta_1]$. Let Θ_f and Θ_g be the unknown cost parameters that have the associated cumulative distribution functions (c.d.f.) $\mathbf{F}(\cdot)$ and $\mathbf{G}(\cdot)$, respectively. Let $\mathbf{f}(\cdot)$ and $\mathbf{g}(\cdot)$ be their respective probability density functions (p.d.f.).

1. First-order stochastic dominance.

A cumulative distribution function $\mathbf{G}(\cdot)$ has first-order stochastic dominance over the cumulative distribution function $\mathbf{F}(\cdot)$ if:

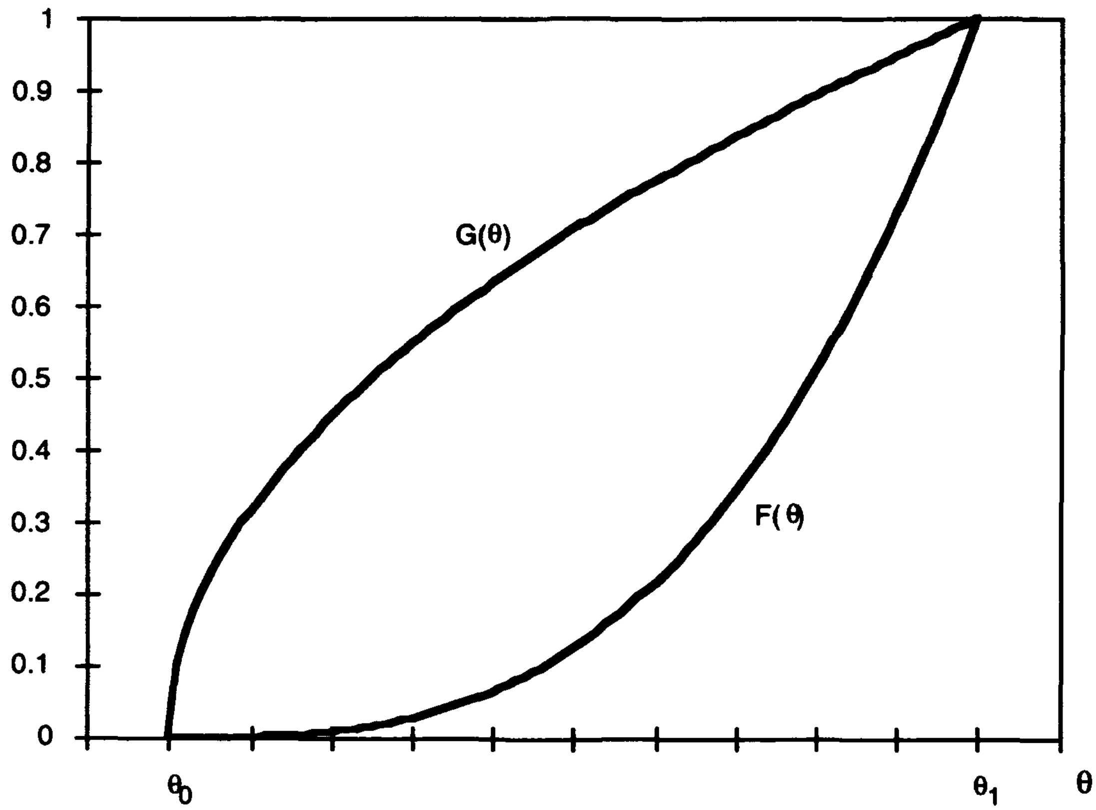
$$\mathbf{G}(\theta) \geq \mathbf{F}(\theta) \qquad \forall \theta \in [\theta_0, \theta_1] \tag{1}$$

To interpret equation (1), recall that $\mathbf{G}(\theta)$ is the probability that $\Theta_g \leq \theta$, Then (1) can be rewritten as:

$$\mathbf{P}(\Theta_g \leq \theta) \geq \mathbf{P}(\Theta_f \leq \theta) \qquad \forall \theta \in [\theta_0, \theta_1] \tag{2}$$

Since $\mathbf{G}(\cdot)$ and $\mathbf{F}(\cdot)$ describe the cumulative probability distribution of cost parameters Θ_g and Θ_f respectively, equation (2) means that the cost parameter associated with $\mathbf{G}(\cdot)$ tends to have lower values than the cost parameter associated with $\mathbf{F}(\cdot)$. Figure 5 depicts the relationship between $F(\cdot)$ and $G(\cdot)$. It is easy to see that $\mathbf{G}(\theta) \geq \mathbf{F}(\theta)$ implies that the probability that Θ_g has a value "near" θ_0 is higher than the probability that Θ_f has a value "near" θ_0. Similarly, the probability that Θ_f has a value "near" θ_1 is higher than the probability that Θ_g has a value "near" θ_1.

FIGURE 5



Example of first-order stochastic dominance of G(·) over F(·).

2. Hazard rate dominance.

A cumulative distribution function $\mathbf{G}(\cdot)$, with its associated probability distribution function $\mathbf{g}(\cdot)$, has hazard rate dominance over the cumulative distribution function $\mathbf{F}(\cdot)$ (and $\mathbf{f}(\cdot)$ respectively) if:

$$\frac{\mathbf{g}(\theta)}{\mathbf{G}(\theta)} \leq \frac{\mathbf{f}(\theta)}{\mathbf{F}(\theta)} \qquad \forall \theta \in [\theta_0, \theta_1] \tag{3}$$

The quotient $\frac{\mathbf{g}(\theta)}{\mathbf{G}(\theta)}$, or the hazard rate, has the following interpretation for the context of this thesis[3]: θ_1 represents the upper bound for the unknown cost parameter, and it is assumed known for both the regulator and the producer. θ_1-θ represents the "cost reduction" achieved by the producer due to its improved technology or efficiency or other factor. $\mathbf{G}(\theta)$ therefore represents the probability that the "cost reduction" is at least θ_1-θ.[4] The probability that there are more than θ_1-θ "cost reductions" and less than θ_1-θ+$d\theta$ "cost reductions" is thus $\mathbf{g}(\theta)d\theta$. Decreasing θ from θ_1, $\mathbf{g}(\theta)/\mathbf{G}(\theta)$ is the conditional probability that there are no more "cost reductions" given that there have already been θ_1-θ "cost reductions". Then (3) requires that the conditional probability that there are no more cost reductions relative to the known maximum cost parameter when the cost reduction is already θ_1-θ is smaller for $\mathbf{G}(\cdot)$ than for $\mathbf{F}(\cdot)$.

If only equation (3) is satisfied, it can be said that $\mathbf{G}(\cdot)$ is ***more favorable*** than $\mathbf{F}(\cdot)$. If both equations (1) and (3) are satisfied, then $\mathbf{G}(\cdot)$ is ***strictly more favorable*** than $\mathbf{F}(\cdot)$. In other words, "...one distribution is [strictly] more favorable than another if it puts more weight on more efficient types," this is, producers that have lower cost parameters. [Ref. 3]

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[3]Adapted from [Ref. 4], pp. 66-67,77.

[4] $P(\Theta^* \geq \theta_1 - \theta) \Leftrightarrow P(\Theta \leq \theta) = \mathbf{G}(\theta)$, where Θ^* is the random variable representing the cost reduction, $\Theta^*=\theta_1-\Theta$.

3. Monotone hazard rate or log-concavity of a cumulative distribution function.

If the c.d.f. associated with the random variable Θ_f satisfies the following condition:

$$\frac{d}{d\theta}\left[\frac{\mathbf{F}(\theta)}{\mathbf{f}(\theta)}\right] \geq 0 \Leftrightarrow \frac{d}{d\theta}\left[\frac{\mathbf{f}(\theta)}{\mathbf{F}(\theta)}\right] \leq 0 \tag{4}$$

Then the function $\mathbf{F}(\cdot)$ is log-concave, and given the previous probabilistic interpretation of the quotient $\mathbf{f}(\theta)/\mathbf{F}(\theta)$, equation (4) states that the conditional probability increases as the firm becomes more efficient. In this sense, this equation represents a decreasing returns condition. This condition is satisfied by most usual distributions: uniform, normal, etc.[5]

[5]This condition is adapted from [Ref. 4], where the assertion that the usual distributions (uniform, normal, etc) comply with it, is made. In their paper, Baron and Myerson do not require the monotone hazard rate condition to be true. However, assuming that this condition is valid, simplifies the mathematical treatment of the problem.

III. ANALYSIS OF THE EFFECT OF MISMATCH BETWEEN REALITY AND BELIEF OF THE REGULATOR.

In Chapter II, the B-M method was explained in qualitative terms. In this chapter, a more rigorous mathematical treatment is developed under the assumption that there is a mismatch between the government's belief about the cost distribution function of the unknown cost parameter and its real cost distribution function. First, the mathematical form of the B-M model is developed as a framework for the subsequent analysis. This is followed by the enunciation of the main theoretical results that predict the behavior of the various measures of welfare that the government may use to determine its policy. Finally, the theory is applied for a specific example, that allows a more intuitive appreciation of the results.

A. MATHEMATICAL FORMULATION OF THE B-M MECHANISM.

The government uses the Baron-Myerson mechanism to determine the pricing and quantity of the product it will procure and the subsidy that the firm will receive. The equations that determine the optimal policy are:

$$z_\alpha(\theta) = \theta + (1-\alpha)\cdot\frac{\mathrm{G}(\theta)}{\mathrm{g}(\theta)} \tag{5}$$

$$p(\theta) = c_0 + c_1 \cdot z_\alpha(\theta) \tag{6}$$

$$q(\theta) = \mathrm{Q}(p(\theta)) \tag{7}$$

$$r(\theta) = \begin{cases} 1 & \text{If } \mathrm{V}(q(\theta)) - p(\theta)\cdot q(\theta) \geq k_0 + k_1 \cdot z_\alpha(\theta) \\ 0 & \text{If } \mathrm{V}(q(\theta)) - p(\theta)\cdot q(\theta) < k_0 + k_1 \cdot z_\alpha(\theta) \end{cases} \tag{8}$$

$$t(\theta) = \left\{(c_0 + c_1\cdot\theta)\cdot q(\theta) + k_0 + k_1\cdot\theta - p(\theta)\cdot q(\theta)\right\}\cdot r(\theta) + \int_\theta^{\theta_1} r(\xi)\cdot\left(c_1\cdot q(\xi) + k_1\right)d\xi \tag{9}$$

Θ is the unknown cost parameter of a contractor, and θ is a particular value that Θ may take. The unknown cost parameter Θ may take values between θ_0 and θ_1, this is, $\theta_0 \leq \theta \leq \theta_1$.

$r(\theta)$ is the probability that the government will permit the contractor to do business at all if it reports a cost parameter of θ. $p(\theta)$ is the price that the government pays for the goods, $q(\theta)$ is the quantity that the government buys at $p(\theta)$ and $t(\theta)$ is the transfer (or subsidy) that the firm receives from the government, all of these values when the firm reports a cost of θ. α is the relative weight given to the contractor's surplus when the government tries to maximize total surplus ($0 \leq \alpha \leq 1$). If $\alpha=1$, we have equal weight among government and producer surplus. If $\alpha=0$, the government only maximizes its own expected gain.

$\mathbf{P}(\cdot)$ is the inverse demand function, and $\mathbf{Q}(\cdot)$ is the demand function. These functions are defined as:

$$\mathbf{Q}(p) = a - b \cdot p \quad \text{and} \quad \mathbf{P}(q) = \frac{a}{b} - \frac{q}{b} \tag{10}$$

and $\mathbf{V}(q)$ is the total value to the government of an output quantity q, defined as:

$$\mathbf{V}(q) = \int_0^q \mathbf{P}(\xi) d\xi = \frac{a}{b} \cdot q - \frac{1}{2b} \cdot q^2 \tag{11}$$

The parameters c_0, c_1, k_0 and k_1 are related to the cost function of the firm. The cost function for producing a quantity q of the goods is:

$$\mathbf{C}(q, \theta) = (c_0 + c_1 \cdot \theta) \cdot q + k_0 + k_1 \cdot \theta \tag{12}$$

Let $\mathbf{f}(\theta)$ be the p.d.f. for Θ, as it occurs in reality, this is, the contractor determines that Θ is distributed according to $\mathbf{f}(\theta)$ from its privileged information about its technology, effort, etc. Let $\mathbf{F}(\theta)$ be its associated c.d.f. Let $\mathbf{g}(\theta)$ be the p.d.f. that the government assigns to Θ, this is, the government believes that Θ is distributed according to $\mathbf{g}(\theta)$. Let $\mathbf{G}(\theta)$ be its associated c.d.f. It will be assumed that $\mathbf{f}(\cdot)$ and $\mathbf{g}(\cdot)$ comply with the monotone hazard rate assumption defined previously. This assumption simplifies the analysis of the Baron-Myerson mechanism.

Let θ^* be the solution to the equation:

$$\mathrm{V}(q(\theta)) - p(\theta)\cdot q(\theta) = k_0 + k_1 \cdot z_\alpha(\theta) \tag{13}$$

Since $r(\theta)$ is a non-increasing function[6] of θ, if $\theta<\theta^*$ then the government allows the firm to do business (i.e., $r(\theta)=1$). Otherwise, the government does not allow the firm to do business and $r(\theta)=0$, $t(\theta)=0$ and $q(\theta)=0$. Therefore, θ^* is a function of a, b, c_0, c_1, k_0, k_1, α , θ_0, θ_1, $\mathbf{g}(\cdot)$ and $\mathrm{G}(\cdot)$.

1. Expression for the expected government gain.

Let $\mathrm{S}(\theta)$ be the government surplus as function of the reported cost parameter θ. Then, the following equation defines $\mathrm{S}(\cdot)$ in terms of the previous equations.

$$\mathrm{S}(\theta) = \mathrm{V}(q(\theta)) - p(\theta)\cdot q(\theta) - t(\theta) \tag{14}$$

and the expected value of this gain, given that the Θ is really distributed according to $\mathbf{f}(\cdot)$, is:

$$E[\mathrm{S}(\theta)] = \int_{\theta_0}^{\theta_1} \mathrm{S}(\xi)\mathbf{f}(\xi)d\xi \tag{15}$$

Since $\mathrm{S}(\theta)$ is equal to zero if $\theta>\theta^*$, then the expected government gain can also be expressed as:

$$E[\mathrm{S}(\theta)] = \int_{\theta_0}^{\theta^*} \mathrm{S}(\xi)\mathbf{f}(\xi)d\xi \tag{16}$$

If $\theta<\theta^*$, then it is possible to get an explicit expression for $\mathrm{S}(\theta)$:

$$\mathrm{S}(\theta) = \frac{a}{b}q - \frac{1}{2b}q^2 - p\cdot q - (c_0 + c_1\theta)\cdot q - k_0 - k_1\theta + p\cdot q - \int_\theta^{\theta^*}(c_1 q(\xi) + k_1)d\xi \tag{17}$$

[6]See [Ref. 2] for other properties of the B-M mechanism.

If equations (5)-(7) are substituted in (17), the following expression for $\mathbf{S}(\theta)$ is derived:

$$\mathbf{S}(\theta)=\mathbf{K}(\theta^*)-\frac{bc_1^2}{2}\left((1-\alpha)^2\frac{\mathbf{G}^2(\theta)}{\mathbf{g}^2(\theta)}-2\cdot(1-\alpha)\int_\theta^{\theta^*}\frac{\mathbf{G}(\xi)}{\mathbf{g}(\xi)}d\xi\right) \tag{18}$$

where

$$\mathbf{K}(\theta^*)=-\left(k_0+k_1\theta^*\right)+\frac{b}{2}\left(\frac{a}{b}-\left(c_0+c_1\theta^*\right)\right)^2 \tag{19}$$

Finally, substituting (18) into (16), we get:

$$E[\mathbf{S}(\theta)]=\mathbf{K}(\theta^*)\int_{\theta_0}^{\theta^*}\mathbf{f}(\xi)d\xi$$

$$-\frac{bc_1^2}{2}\int_{\theta_0}^{\theta^*}\left((1-\alpha)^2\frac{\mathbf{G}^2(\xi)}{\mathbf{g}^2(\xi)}-2\cdot(1-\alpha)\int_\xi^{\theta^*}\frac{\mathbf{G}(\zeta)}{\mathbf{g}(\zeta)}d\zeta\right)\mathbf{f}(\xi)d\xi \tag{20}$$

Note that:

$$\int_{\theta_0}^{\theta^*}\left(\int_{\xi=\zeta}^{\theta^*}\frac{\mathbf{G}(\xi)}{\mathbf{g}(\xi)}d\xi\right)\mathbf{f}(\zeta)d\zeta=\int_{\theta_0}^{\theta^*}\frac{\mathbf{G}(\xi)}{\mathbf{g}(\xi)}\left(\int_{\theta=\theta_0}^{\xi}\mathbf{f}(\zeta)d\zeta\right)d\xi=\int_{\theta_0}^{\theta^*}\frac{G(\xi)}{g(\xi)}\frac{F(\xi)}{f(\xi)}f(\xi)d\xi \tag{21}$$

and let

$$\theta_c=\begin{cases}\theta_1 & \text{If } \theta^*\geq\theta_1\\ \theta^* & \text{If } \theta^*<\theta_1\end{cases} \tag{22}$$

Then

$$E[\mathbf{S}(\theta)]=\mathbf{K}(\theta_c)\mathbf{F}(\theta_c)$$

$$-bc_1^2(1-\alpha)\int_{\theta_0}^{\theta_c}\left(\frac{(1-\alpha)}{2}\frac{\mathbf{G}(\xi)}{\mathbf{g}(\xi)}-\frac{\mathbf{F}(\xi)}{\mathbf{f}(\xi)}\right)\frac{\mathbf{G}(\xi)}{\mathbf{g}(\xi)}\mathbf{f}(\xi)d\xi \tag{23}$$

For simplicity of the analysis, two cases will be considered:

(a) Unknown variable cost and known fixed cost: In this case, $c_0=0$, $c_1=1$, $k_0=k_0$ and $k_1=0$. In this case, the cost parameter Θ represents the unknown marginal cost, while the government knows the fixed cost.

For this case, (13) is simplified to:

$$q(\theta^*)=\sqrt{2k_0b}\Leftrightarrow\frac{a}{b}-\theta^*=\sqrt{\frac{2k_0}{b}}+(1-\alpha)\frac{\mathbf{G}(\theta^*)}{\mathbf{g}(\theta^*)} \tag{24}$$

and $\mathbf{K}(\theta^*)$ is now:

$$\mathbf{K}(\theta^*) = -k_0 + \frac{b}{2}\left(\frac{a}{b} - \theta^*\right)^2 = \frac{(1-\alpha)b}{2}\frac{G(\theta^*)}{g(\theta^*)}\left((1-\alpha)\frac{G(\theta^*)}{g(\theta^*)} + 2\sqrt{\frac{2k_0}{b}}\right) \quad (25)$$

(b) Unknown fixed cost and known variable cost: In this case, $c_0 = c_0$, $c_1 = 0$, $k_0 = 0$ and $k_1 = 1$. In this case, the cost parameter θ represents the unknown fixed cost, while the government knows the marginal cost.

For this simple case, the expected consumer gain is equal to:

$$E[\mathbf{S}(\theta)] = \mathbf{K}(\theta_c)\mathbf{F}(\theta_c) \quad (26)$$

where

$$\mathbf{K}(\theta_c) = -\theta_c + \frac{b}{2}\left(\frac{a}{b} - c_0\right)^2 \quad (27)$$

Unless specifically mentioned, the rest of the sections will refer to the first case, the unknown marginal cost and known fixed cost.

2. Expression for the firm's expected profit.

The profit for the producer is:

$$\pi(\theta) = p(\theta) \cdot q(\theta) - \mathbf{C}(q,\theta) + t(\theta) \quad (28)$$

$$\pi(\theta) = \int_{\theta}^{\theta_c} q(\xi)d\xi \Rightarrow \dot{\pi}(\theta) = -q(\theta) \quad (29)$$

And the expected profit is:

$$E[\pi(\theta)] = \int_{\theta_0}^{\theta_c} \pi(\xi)\mathbf{f}(\xi)d\xi = \pi(\xi)\mathbf{F}(\xi)\Big|_{\xi=\theta_0}^{\xi=\theta_c} - \int_{\theta_0}^{\theta_c} \dot{\pi}(\xi)\frac{\mathbf{F}(\xi)}{\mathbf{f}(\xi)}\mathbf{f}(\xi)d\xi \quad (30)$$

The second part of the equality is obtained after integrating by parts. Since $\mathbf{F}(\theta_0)=0$ and $\pi(\theta_c) = 0$, then:

$$E[\pi(\theta)] = \int_{\theta_0}^{\theta_c} (-\dot{\pi}(\xi))\frac{\mathbf{F}(\xi)}{\mathbf{f}(\xi)}\mathbf{f}(\xi)d\xi = \int_{\theta_0}^{\theta_c} q(\xi)\frac{\mathbf{F}(\xi)}{\mathbf{f}(\xi)}\mathbf{f}(\xi)d\xi \quad (31)$$

B. IMPACT ON DIFFERENT MEASUREMENTS OF WELFARE WHEN THE GOVERNMENT VARIES THE CUMULATIVE DISTRIBUTION FUNCTION.

It will be assumed now that the government does not know the real distribution function for the cost parameter. In this section, the impact on the different measurements that the government can use to guide its policy will be studied as a function of the selected distribution, while the unknown real distribution remains the same.

A family of distributions is used to model the availability of different choices for the selection of a distribution function for the unknown cost parameters. Let $\mathbf{H}(\theta,\rho)$ be a cumulative distribution function family with the following properties:

(1) $\mathbf{H}(\theta,\rho)$ is log-concave for any ρ: $\dfrac{\partial}{\partial\theta}\left[\dfrac{\mathbf{H}(\theta,\rho)}{\mathbf{h}(\theta,\rho)}\right]\geq 0$, where $\mathbf{h}(\theta,\rho)=\dfrac{\partial\mathbf{H}(\theta,\rho)}{\partial\theta}$

(2) $\dfrac{\partial}{\partial\rho}\left[\dfrac{\mathbf{H}(\theta,\rho)}{\mathbf{h}(\theta,\rho)}\right]\geq 0 \Leftrightarrow \dfrac{\partial}{\partial\rho}\left[\dfrac{\mathbf{h}(\theta,\rho)}{\mathbf{H}(\theta,\rho)}\right]\leq 0$

$\Leftrightarrow \rho_2\geq\rho_1 \Rightarrow \dfrac{\mathbf{H}(\theta,\rho_2)}{\mathbf{h}(\theta,\rho_2)}\geq\dfrac{\mathbf{H}(\theta,\rho_1)}{\mathbf{h}(\theta,\rho_1)}\ \forall\theta\in[\theta_0,\theta_1]$, this is, as ρ increases, the cumulative function has greater hazard rate dominance.

(3) $\dfrac{\partial\mathbf{H}(\theta,\rho)}{\partial\rho}\geq 0 \Leftrightarrow \rho_2\geq\rho_1 \Rightarrow \mathbf{H}(\theta,\rho_2)\geq\mathbf{H}(\theta,\rho_1)\ \forall\theta\in[\theta_0,\theta_1]$, this is, as ρ increases, the cumulative function has greater first-order stochastic dominance.

(4) $\mathbf{H}(\theta,\rho_M)=\mathbf{F}(\theta)\ \forall\theta\in[\theta_0,\theta_1]$, this is, $\mathbf{H}(\cdot,\rho)$ is identical to $\mathbf{F}(\cdot)$ when $\rho=\rho_M$. Since $\mathbf{F}(\cdot)$ is unknown, ρ_M is also unknown.

Properties (1) through (4) describe a family of functions whose degree of "favorability" is measured with the variable ρ. Also, the real (and unknown) distribution function $\mathbf{F}(\cdot)$ is a member of this family of functions. The government uses $\mathbf{G}(\theta)=\mathbf{H}(\theta,\rho)$ as the selected distribution function to apply its purchasing policy.

The government may use different criteria to select a specific policy represented by ρ:

(1) Select a distribution that maximizes expected government gain $E[S(\theta,\rho)]$, this is, select the value of ρ that maximizes $E[S(\theta,\rho)]$, without giving any weight to the firm's profit. This criterion requires that $\alpha=0$.

(2) Select a distribution that maximizes the weighted sum of government gain and producer's profit $E[W]=E[S]+\alpha E[\pi]$, where α is defined in the mathematical formulation of the B-M mechanism. This is equivalent to say that the government will select the value of ρ that maximizes $E[W]=E[S]+\alpha E[\pi]$.

(3) Select a distribution that maximizes the simple sum of government gain and producer's profit $E[W]=E[S]+E[\pi]$.

For the procurement case, the first criteria is used because the government is interested in maximizing its own gain. However, it is still important to understand what implications does the use of this criteria have on both the government and the firm.

The following theorems summarize the main results of this thesis. The demonstrations are presented at the end of this chapter.

Theorem 1: *For any value of α, the expected total welfare $E[W]=E[S]+\alpha E[\pi]$ has a global maximum at $\rho=\rho_M$.*

Corollary: *If $\alpha=0$, the expected government gain has a global maximum at $\rho=\rho_M$.*

Theorem 2: *For any value of α, the expected total welfare $E[W]=E[S]+E[\pi]$ is a non-increasing function of ρ.*

Theorem 3: *For any value of α, the firm's expected profit is a non-increasing function of ρ.*

Theorem 4: *The expected government gain tends to zero as ρ increases, when $\rho>\rho_M$. If $\rho<\rho_M$, $\lim_{\rho\to-\infty}(H(\theta,\rho)/h(\theta,\rho))=0$ and the fixed cost is small, then the expected government gain tends to a value higher than zero as ρ decreases.*

These theorems have very important policy implications for the use of the Baron-Myerson mechanism in the defense procurement arena.

If the government uses criterion (1) to select the distribution function for the cost parameter, theorem's 1 corollary implies that the best outcome occurs when the government has perfect information about the real distribution of the cost parameter, this is, the government is able to determine how Θ is distributed although it still does not know the real value of Θ. If the government chooses a distribution function that is less favorable than the real distribution function and then it decides to increase its "favorability", then the expected government gain will increase. On the other hand, if the government chooses a distribution function that is more favorable than the real distribution function, and then it decides to increase its "favorability", then the expected government gain will decrease. This behavior, together with theorem 4, implies that the government may be better off by choosing the distribution function "conservatively" (i.e., choose relatively less favorable distributions). If it pushes the "favorability" of the distribution function over a certain limit (if it pursues an "aggressive" policy), its expected gain may be lower than if it chooses a "less favorable" distribution. This interpretation of theorem's 1 corollary also applies to theorem 1 itself if the government uses criterion 2 to select the probability distribution.

Theorem 2 is interpreted as follows: as the "favorability" of the distribution function increases, the overall expected welfare decreases. Theorem 2 can also be interpreted in terms of a deadweight loss. If $\rho=\rho_1$ defines a baseline for welfare measurement, then any the choice of $\rho>\rho_1$ diminishes total expected welfare, or, increases expected deadweight loss.

Theorem 2 has also important implications for policy making. If the government is uncertain about the real distribution and has risk aversion, it may decide to choose a distribution function that is intentionally less favorable in relative terms, because it would increase the overall expected welfare, though at the cost of its own gain. Other exogenous factors may also push in this direction of being "conservative" about the efficiency of the firm, like job security, industry subsidies, etc.

Theorem 3 has important implications for the choice of c.d.f. and the incentive structure that the B-M mechanism configures for the producer. First, the choice of a more favorable c.d.f. reduces the rent of the producer, therefore the government may use this procedure as a rent extraction tool. Unfortunately, this cannot be applied indefinitely, because after a certain limit the government's actions in this dimension would also reduce its own surplus.

Second, the producer has an incentive to lie about its true cost's c.d.f. when it tries to influence the regulator. If the producer is successful, the government will choose a cost distribution function that is less favorable than the real cost distribution function. Then, the producer will profit more than if the government used the real cost distribution function.

Third, if $\alpha=0$ (no weight to the producer's profit in the maximization problem), the incentive for the producer to behave strategically (i.e., lobbying effort) is greater, measured in terms of the impact of a successful influence in the expected profit (See equation (51) in the demonstration of theorem 3, below).

Summarizing, the form of an optimal policy will depend on which criterion the government uses to select the distribution function. If it uses criterion 1 or 2, the mechanism would yield the best outcome when the information about the real distribution function is complete (i.e., the regulator is able to determine $\mathbf{F}(\theta)$). If it uses criterion 3, the government has an incentive to use "less efficient" distributions.

Now, if the first-best alternative is not achievable because of insufficient information about the cost distribution (i.e. the regulator is not able to determine $\mathbf{F}(\theta)$), the second-best alternative would depend on how expected surplus (or welfare) behaves on either side of the optimum. The possibility of not doing business at all (because of the requirement of extreme efficiency) suggests that the overall expected welfare will eventually decrease sharply as the regulator increases the "favorability" of the chosen c.d.f. On the other hand, decreasing too much the "favorability" of the chosen c.d.f. puts in question the effectiveness of the policy, and the government may be better off not using this mechanism at all when there is a great uncertainty about the firm's costs, and instead using the Loeb-Magat mechanism, that is much simpler to carry out. The following example should clarify these options, although there will be a loss of generality.

C. EXAMPLE OF THE IMPACT OF C.D.F. CHOICE.

Consider the family of c.d.f.'s:

$$\mathbf{H}(\theta,\rho)=\left(\frac{\theta-\theta_0}{\theta_1-\theta_0}\right)^{(1-\rho)} \qquad \rho<1,\theta_0\le\theta\le\theta_1 \tag{32}$$

Clearly, $\mathbf{H}(\theta_0,\rho)=0$ and $\mathbf{H}(\theta_1,\rho)=1$. Then,

$$\mathbf{h}(\theta,\rho)=\frac{1-\rho}{\theta_1-\theta_0}\left(\frac{\theta-\theta_0}{\theta_1-\theta_0}\right)^{-\rho} \tag{33}$$

and

$$\frac{\mathbf{H}(\theta,\rho)}{\mathbf{h}(\theta,\rho)}=\frac{\theta-\theta_0}{1-\rho} \tag{34}$$

This c.d.f. satisfies the properties described before:

$$\frac{\partial\mathbf{H}(\theta,\rho)}{\partial\rho}=-\left(\frac{\theta-\theta_0}{\theta_1-\theta_0}\right)^{1-\rho}\log\left(\frac{\theta-\theta_0}{\theta_1-\theta_0}\right)\ge 0 \quad \text{because} \left(\frac{\theta-\theta_0}{\theta_1-\theta_0}\right)\le 1 \tag{35}$$

$$\frac{\partial}{\partial\rho}\left[\frac{\mathbf{H}(\theta,\rho)}{\mathbf{h}(\theta,\rho)}\right]=\frac{\theta-\theta_0}{(1-\rho)^2}\ge 0 \tag{36}$$

$$\frac{\partial}{\partial\theta}\left[\frac{\mathbf{H}(\theta,\rho)}{\mathbf{h}(\theta,\rho)}\right]=\frac{1}{1-\rho}\ge 0 \tag{37}$$

Lets assume that the real c.d.f is:

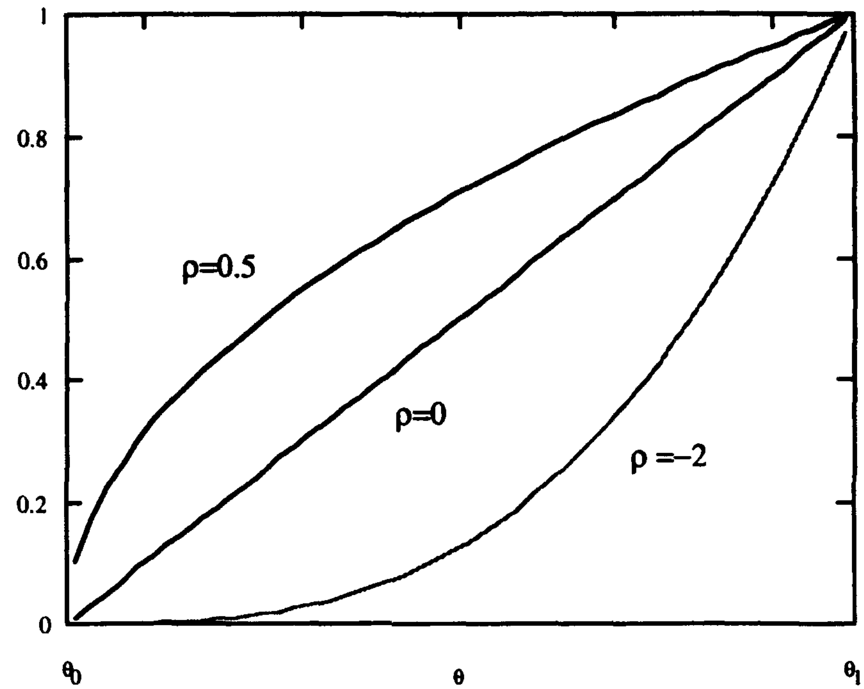
$$\mathbf{F}(\theta)=\frac{\theta-\theta_0}{\theta_1-\theta_0}=\mathbf{H}(\theta,\rho=0) \tag{38}$$

Therefore, if $\rho>0$, $\mathbf{H}(\theta,\rho)$ is more favorable than $\mathbf{F}(\theta)$. Also, if $\rho<0$, $\mathbf{H}(\theta,\rho)$ is less favorable than $\mathbf{F}(\theta)$. In this case, $\rho_M=0$. $\mathbf{F}(\theta)$ is an uniform distribution, this is, the probability that the cost takes a value between θ and $\theta+d\theta$ is constant for any θ. If $\rho>0$, the probability density function is skewed toward the lower bound of θ. If $\rho<0$, the probability density function is skewed toward the upper bound of θ. Note that for $\rho>0$, the slope of the p.d.f. is negative, while if $\rho<0$, the slope of the p.d.f. is positive.

$\mathbf{H}(\theta,\rho)$ may represent the cost distribution associated with an "efficient" firm ($\rho>0$), an "inefficient" firm ($\rho<0$) or an "average" firm ($\rho=0$), and therefore is useful for the comparison of different "types" of firms. The following figures (Figures 6 and 7) illustrate $\mathbf{H}(\theta,\rho)$ and $\mathbf{h}(\theta,\rho)$ for $\rho=0.5$, 0 and -2.

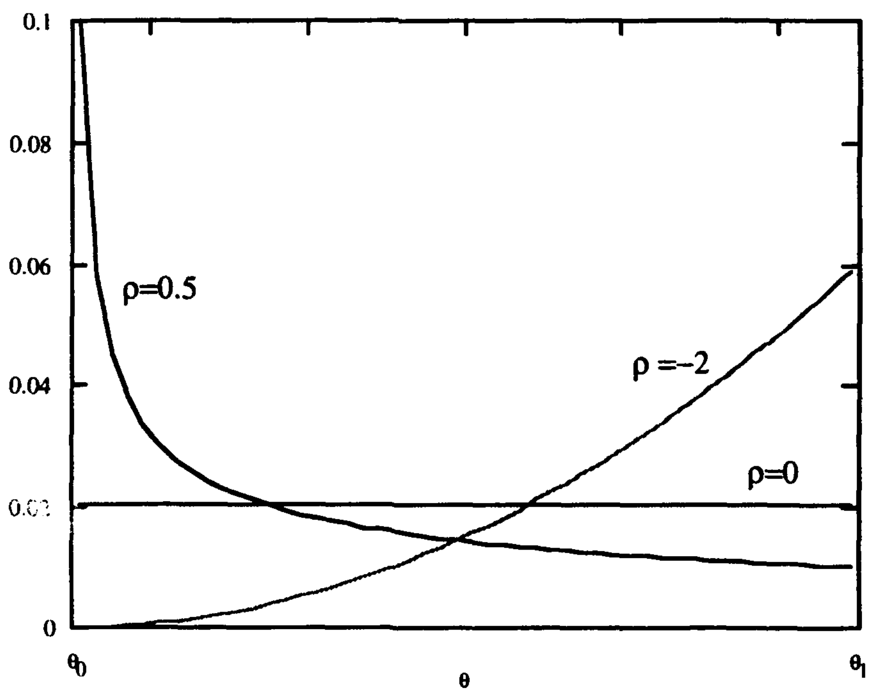
Since $\mathbf{H}(\theta,\rho)$ is equal to $\mathbf{F}(\theta)$ when $\rho=0$, $\partial E[\mathbf{W}]/\partial\rho=0$ at $\rho=0$, and the total welfare is maximized. Theorem's 1 corollary implies that $\partial E[\mathbf{S}(\theta,\rho)]/\partial\rho=0$ at $\rho=0$, and a local maximum is obtained for the expected government gain. Figures 8 and 9 show the expected total welfare, expected government gain and expected producer's profit for $\alpha=0$ as a function of ρ.

FIGURE 6



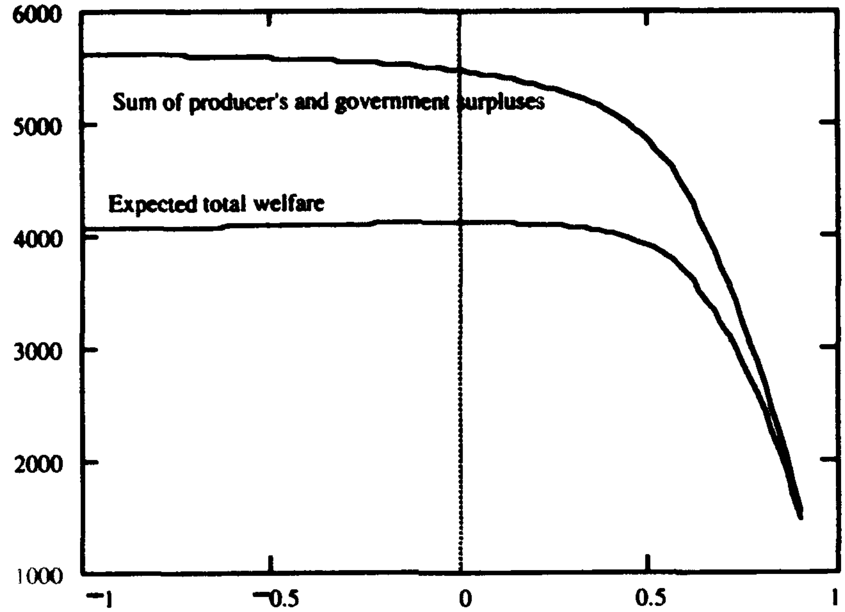
Cumulative distribution function H(θ,ρ) for different values of ρ.

FIGURE 7



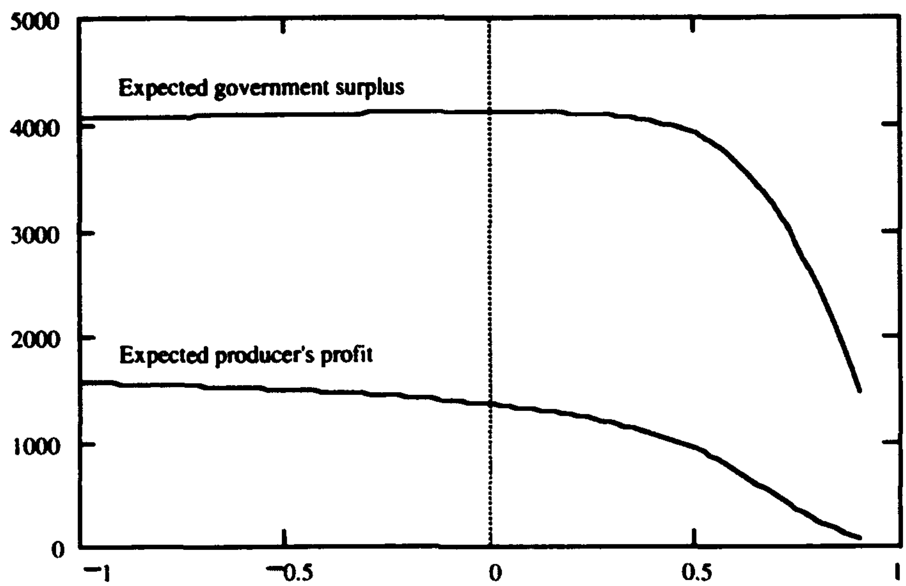
Probability density function h(θ,ρ) for different values of ρ.

FIGURE 8



Expected total welfare and sum of producer and government welfare for α=0 as a function of ρ

FIGURE 9

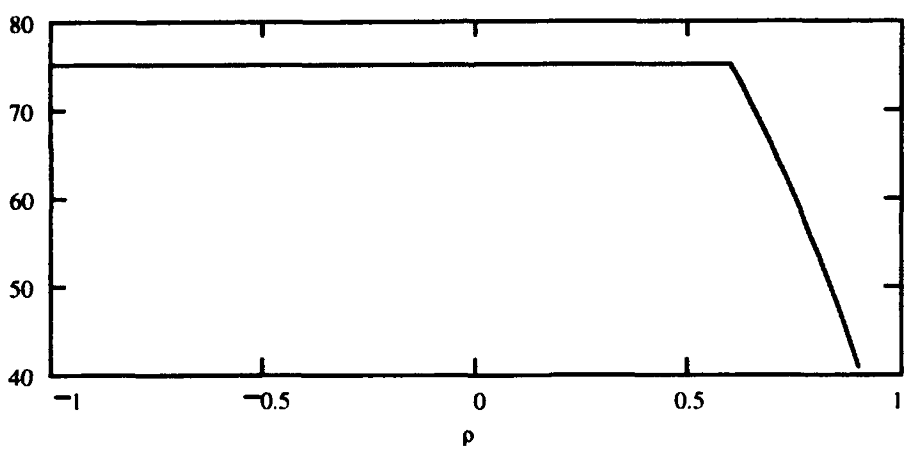


Expected consumer's surplus and expected producer's profit for α=0 as a function of ρ

For this example, the demand curve is $Q(P)=100-0.5\cdot P$ and the fixed cost is zero. The cost parameter Θ varies between 25 and 75. As it was predicted by theorem 2 and theorem's 1 corollary, the slope of the expected profit is always negative and the expected consumer surplus is maximized at $\rho=0$.

The next figure (Figure 10) shows the cutoff value for the cost parameter as a function of ρ. It can be clearly seen that after a certain value of ρ, the government sometimes shuts off the producer because its cost parameters are too high. This is consistent with the choice of c.d.f. Higher values of ρ indicate that the regulator is interested in companies with low cost parameters. Then, it is expected that the regulator will eventually stop production if it requires more and more efficiency. As ρ approaches 1, the regulator's cost distribution approaches an impulse function at $\theta=\theta_0$, and it will allow a business to produce only if $\theta=\theta_0$ and it will extract all of the firm's rent, this is, the profit will be zero.

FIGURE 10



Cutoff value of the cost parameter (θ_c)as a function of ρ.

Consider now the limit situation when $\rho \to -\infty$. In this case:

$$\lim_{\rho\to-\infty}\left(\mathbf{H}(\theta,\rho)/\mathbf{h}(\theta,\rho)\right)=0$$

$$\lim_{\rho\to-\infty}\left(\mathbf{S}(\theta,\rho)\right)=-k_0+\frac{b}{2}\left(\frac{a}{b}-\theta_1\right)^2=3906.25$$

$$\lim_{\rho\to-\infty}\left(E[\mathbf{S}(\theta,\rho)]\right)=-k_0+\frac{b}{2}\left(\frac{a}{b}-\theta_1\right)^2=3906.25 \tag{39}$$

Let us compare these equations with the monopoly outcome. If the regulator decides not to intervene, the firm will maximize its profits:

$$\underset{q}{\mathbf{Max}}(\pi) = p \cdot q - \mathrm{C}(q) \Rightarrow \frac{d\pi}{dq} = 0 \Rightarrow p + p' \cdot q - \mathrm{C}'(q) = 0 \tag{40}$$

Using the demand curve and the cost function, equation (40) implies that for the monopoly outcome:

$$\mathrm{S}(\theta) = \frac{b}{8}\left(\frac{a}{b} - \theta\right)^2$$

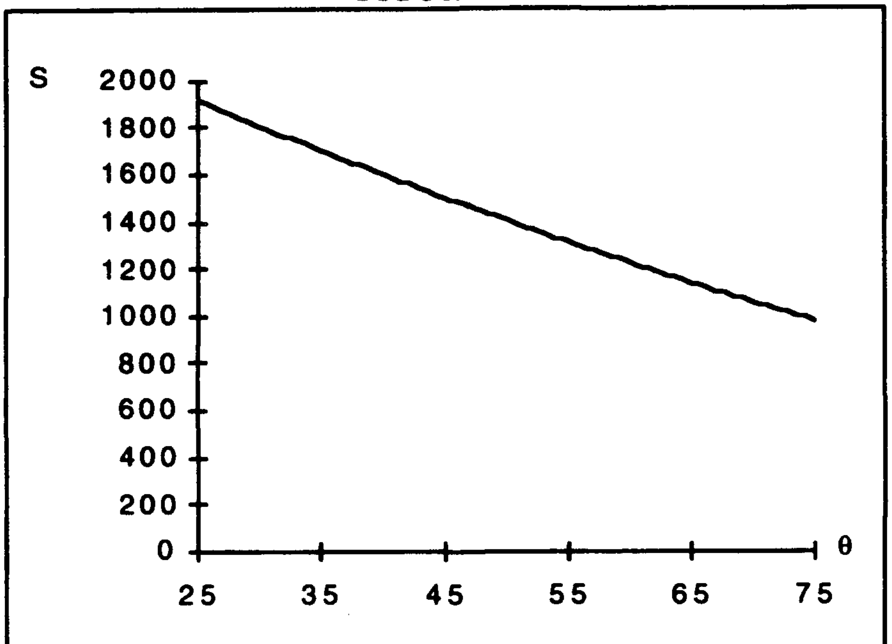
$$E[\mathrm{S}(\theta)] = \int_{\theta_0}^{\theta_1} \frac{b}{8}\left(\frac{a}{b} - \xi\right)^2 \mathrm{f}(\xi) d\xi = \frac{b}{24} \frac{\left(\frac{a}{b} - \theta_0\right)^3 - \left(\frac{a}{b} - \theta_1\right)^3}{(\theta_1 - \theta_0)} = 1419.27 \tag{41}$$

Figure 9 shows the expected government gain as a function of the firm's cost parameter. From this figure it is clear that the regulator is better off using a very "unfavorable" cost distribution than letting the firm act as a monopolist. On the other hand, this figure also shows that if the government uses a "very favorable" cost distribution function, its expected gain will drop sharply after reaching the maximum. There will be a value of ρ where the expected government gain using the B-M mechanism will be equal to the expected government gain for the monopolist situation. Let us call this value ρ_c. If the government chooses $\rho > \rho_c$, then it will be better off if it does not use the B-M mechanism.

This result is consistent with theorem 4 and the previous discussion about the second-best choice of c.d.f., given the fact that the real distribution function is unknown. Equations (39) and (41) and Figure 8 suggest that the regulator may be better off choosing in a "conservative" way, this is, assuming a less efficient firm, than choosing in very "optimistic" or "aggressive" way, because eventually the government will be better off if it does not use the B-M model.

The condition $\lim_{\rho \to -\infty} (\mathbf{H}(\theta,\rho)/\mathbf{h}(\theta,\rho)) = 0$ imposes strong restrictions for the selection of $\mathbf{H}(\theta,\rho)$. However, if it holds in addition to the properties mentioned before for $\mathbf{H}(\theta,\rho)$ and the fixed cost is small, then the second-best choice for the government would be to choose a relatively "less efficient" cost distribution function, if the real cost distribution is unknown. Otherwise, the government incurs the risk of choosing a c.d.f. that would eventually close down the firm and the expected consumer surplus will be lower than the expected consumer surplus in the monopoly case.

FIGURE 11



Government gain for the monopoly outcome as a function of the reported cost parameter.

D. PROOF OF THE THEOREMS.

1. Proof of corollary of theorem 1.

To demonstrate that has an optimum at $\rho=\rho_M$, it will be shown that the derivative of the expected government gain with respect to ρ is equal to zero at $\rho=\rho_M$, and that this derivative is always non-positive for $\rho>\rho_M$ and always non-negative for $\rho<\rho_M$. To obtain an expression of $E[\mathbf{S}]$ as a function of ρ, $\mathbf{G}(\cdot)$ is replaced with $\mathbf{H}(\cdot,\rho)$ in (23), and knowing that $\theta_c = \theta_c(\rho)$, then:

$$E[\mathbf{S}(\theta,\rho)] = \mathbf{K}(\theta_c(\rho))\mathbf{F}(\theta_c(\rho)) - b(1-\alpha)\int_{\theta_0}^{\theta_c(\rho)}\left(\frac{(1-\alpha)}{2}\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)} - \frac{\mathbf{F}(\xi)}{\mathbf{f}(\xi)}\right)\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)}\mathbf{f}(\xi)d\xi \tag{42}$$

Taking the partial derivative of (42):

$$\begin{aligned}\frac{\partial E[\mathbf{S}(\theta,\rho)]}{\partial\rho} &= \frac{\partial}{\partial\rho}[\mathbf{K}(\theta_c(\rho))\mathbf{F}(\theta_c(\rho))] \\ &- b(1-\alpha)\int_{\theta_0}^{\theta_c(\rho)}\frac{\partial}{\partial\rho}\left[\left(\frac{(1-\alpha)}{2}\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)} - \frac{\mathbf{F}(\xi)}{\mathbf{f}(\xi)}\right)\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)}\right]\mathbf{f}(\xi)d\xi \\ &- b(1-\alpha)\left(\frac{(1-\alpha)}{2}\frac{\mathbf{H}(\theta_c(\rho),\rho)}{\mathbf{h}(\theta_c(\rho),\rho)} - \frac{\mathbf{F}(\theta_c(\rho))}{\mathbf{f}(\theta_c(\rho))}\right)\frac{\mathbf{H}(\theta_c(\rho),\rho)}{\mathbf{h}(\theta_c(\rho),\rho)}\mathbf{f}(\theta_c(\rho))\frac{d\theta_c(\rho)}{d\rho}\end{aligned} \tag{43}$$

$$\begin{aligned}\frac{\partial E[\mathbf{S}(\theta,\rho)]}{\partial\rho} &= \mathbf{f}(\theta_c(\rho))\frac{\partial\theta_c(\rho)}{\partial\rho}\left[-k_0 + \frac{b}{2}\left(\frac{a}{b}-\theta_c(\rho)\right)^2 - b\left(\frac{a}{b}-\theta_c(\rho)\right)\frac{\mathbf{F}(\theta_c(\rho))}{\mathbf{f}(\theta_c(\rho))}\right] \\ &- b(1-\alpha)\int_{\theta_0}^{\theta_c(\rho)}\frac{\partial}{\partial\rho}\left[\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)}\right]\left((1-\alpha)\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)} - \frac{\mathbf{F}(\xi)}{\mathbf{f}(\xi)}\right)\mathbf{f}(\xi)d\xi \\ &- b(1-\alpha)\left(\frac{(1-\alpha)}{2}\frac{\mathbf{H}(\theta_c(\rho),\rho)}{\mathbf{h}(\theta_c(\rho),\rho)} - \frac{\mathbf{F}(\theta_c(\rho))}{\mathbf{f}(\theta_c(\rho))}\right)\frac{\mathbf{H}(\theta_c(\rho),\rho)}{\mathbf{h}(\theta_c(\rho),\rho)}\mathbf{f}(\theta_c(\rho))\frac{d\theta_c(\rho)}{d\rho}\end{aligned} \tag{44}$$

Using (24), the final expression for $\partial E[\mathbf{S}(\theta,\rho)]/\partial\rho$ is:

$$\frac{\partial E[\mathbf{S}(\theta,\rho)]}{\partial\rho} = \sqrt{2k_0 b}\,\mathbf{f}(\theta_c(\rho))\frac{\partial\theta_c(\rho)}{\partial\rho}\left((1-\alpha)\frac{\mathbf{H}(\theta_c(\rho),\rho)}{\mathbf{h}(\theta_c(\rho),\rho)} - \frac{\mathbf{F}(\theta_c(\rho))}{\mathbf{f}(\theta_c(\rho))}\right)$$
$$- b(1-\alpha)\int_{\theta_0}^{\theta_c(\rho)}\frac{\partial}{\partial\rho}\left[\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)}\right]\left((1-\alpha)\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)} - \frac{\mathbf{F}(\xi)}{\mathbf{f}(\xi)}\right)\mathbf{f}(\xi)d\xi \quad (45)$$

Taking the total differential of (13), and solving for $d\theta_c(\rho)/d\rho$ (assuming that $\theta^* < \theta_1$), the following expression is obtained:

$$\frac{d\theta_c(\rho)}{d\rho} = -\frac{(1-\alpha)\dfrac{\partial}{\partial\rho}\left[\dfrac{\mathbf{H}(\theta,\rho)}{\mathbf{h}(\theta,\rho)}\right]_{\theta=\theta_c(\rho)}}{1+(1-\alpha)\dfrac{\partial}{\partial\theta}\left[\dfrac{\mathbf{H}(\theta,\rho)}{\mathbf{h}(\theta,\rho)}\right]_{\theta=\theta_c(\rho)}} \le 0 \quad (46)$$

(Note that this result for $d\theta_c(\rho)/d\rho$ is valid in the more general case where $c_0\neq 0$, $c_1\neq 1$, $k_1\neq 0$.)

Equation (46) implies that a sufficient condition to determine the sign of $\partial E[\mathbf{S}(\theta,\rho)]/\partial\rho$ is to determine the sign of $(1-\alpha)(\mathbf{H}(\theta,\rho)/\mathbf{h}(\theta,\rho)) - \mathbf{F}(\theta)/\mathbf{f}(\theta)$ for all values of θ. If $(1-\alpha)(\mathbf{H}(\theta,\rho)/\mathbf{h}(\theta,\rho)) - \mathbf{F}(\theta)/\mathbf{f}(\theta) \le 0$ for all θ, then $\partial E[\mathbf{S}(\theta,\rho)]/\partial\rho \ge 0$. If $\alpha=0$, then for $\rho>\rho_M$, $(1-\alpha)(\mathbf{H}(\theta,\rho)/\mathbf{h}(\theta,\rho)) - \mathbf{F}(\theta)/\mathbf{f}(\theta) \le 0$ ($\partial E[\mathbf{S}(\theta,\rho)]/\partial\rho \ge 0$), for $\rho<\rho_M$, $(1-\alpha)(\mathbf{H}(\theta,\rho)/\mathbf{h}(\theta,\rho)) - \mathbf{F}(\theta)/\mathbf{f}(\theta) \ge 0$ ($\partial E[\mathbf{S}(\theta,\rho)]/\partial\rho \le 0$) and for $\rho=\rho_M$, $(1-\alpha)(\mathbf{H}(\theta,\rho)/\mathbf{h}(\theta,\rho)) - \mathbf{F}(\theta)/\mathbf{f}(\theta) = 0$ thus $\partial E[\mathbf{S}(\theta,\rho)]/\partial\rho = 0$. These three conditions are sufficient and necessary for $E[\mathbf{S}(\theta,\rho)]$ to have a maximum at $\rho=\rho_M$ when $\alpha=0$.❑

For the unknown fixed cost case (case (b)), a similar result is reached. In this scenario, $\partial E[\mathbf{S}(\theta,\rho)]/\partial\rho$ is:

$$\frac{\partial E[\mathbf{S}(\theta,\rho)]}{\partial\rho} = \mathbf{f}(\theta_c(\rho))\frac{\partial\theta_c(\rho)}{\partial\rho}\left((1-\alpha)\frac{\mathbf{H}(\theta_c(\rho),\rho)}{\mathbf{h}(\theta_c(\rho),\rho)} - \frac{\mathbf{F}(\theta_c(\rho))}{\mathbf{f}(\theta_c(\rho))}\right) \quad (47)$$

Again, it is clear that if $\alpha=0$, $\partial E[\mathbf{S}(\theta,\rho)]/\partial\rho=0$ when $\rho=\rho_M$ and $E[\mathbf{S}(\theta,\rho)]$ is maximized. ❑

2. Proof of theorem 3.

The demonstration of theorem 3 follows the same reasoning that the demonstration of theorem's 1 corollary. The expected profit as a function of ρ is:

$$E[\pi(\theta,\rho)] = \int_{\theta_0}^{\theta_c(\rho)} (-\dot{\pi}(\xi,\rho))\frac{\mathbf{F}(\xi)}{\mathbf{f}(\xi)}\mathbf{f}(\xi)d\xi \tag{48}$$

Taking the derivative of $E[\pi(\theta,\rho)]$:

$$\frac{\partial}{\partial\rho}[E[\pi(\theta,\rho)]] = \int_{\theta_0}^{\theta_c} -\frac{\partial\dot{\pi}(\xi,\rho)}{\partial\rho}\frac{\mathbf{F}(\xi)}{\mathbf{f}(\xi)}\mathbf{f}(\xi)d\xi + (-\dot{\pi}(\theta_c,\rho))F(\theta_c)\frac{d\theta_c}{d\rho} \tag{49}$$

Since $q(\theta_c,\rho) = \sqrt{2k_0 b}$ (if $\theta^* < \theta_l$) and:

$$\frac{\partial\dot{\pi}}{\partial\rho} = -\frac{\partial q}{\partial\rho} = b(1-\alpha)\frac{\partial}{\partial\rho}\left[\frac{\mathbf{H}(\theta,\rho)}{\mathbf{h}(\theta,\rho)}\right] \tag{50}$$

Then:

$$\frac{\partial}{\partial\rho}[E[\pi(\theta,\rho)]] = -b(1-\alpha)\int_{\theta_0}^{\theta_c}\frac{\partial}{\partial\rho}\left[\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)}\right]\frac{\mathbf{F}(\xi)}{\mathbf{f}(\xi)}\mathbf{f}(\xi)d\xi + \sqrt{2k_0 b}\cdot\mathbf{F}(\theta_c)\frac{d\theta_c}{d\rho} \tag{51}$$

Therefore, $\partial E[\pi(\theta,\rho)]/\partial\rho \le 0$, because of (46) and property (2) of $\mathbf{H}(\theta,\rho)$.❑

3. Proof of theorem 1.

To prove theorem one it is necessary to show that:

$$E[\mathbf{W}] = E[\mathbf{S}] + \alpha E[\pi] \tag{52}$$

Has a maximum at $\rho=\rho_M$. Taking the derivative of E[W] with respect to ρ :

$$\frac{\partial E[\mathbf{W}]}{\partial\rho} = \frac{\partial E[\mathbf{S}]}{\partial\rho} + \alpha\cdot\frac{\partial E[\pi]}{\partial\rho} \tag{53}$$

Using equations (45) and (51), the following expression is obtained for $\partial E[\mathbf{W}]/\partial\rho$:

$$\frac{\partial E[\mathbf{W}]}{\partial\rho}=\sqrt{2k_0b}\,\mathbf{f}(\theta_c(\rho))\frac{\partial\theta_c(\rho)}{\partial\rho}\left[(1-\alpha)\frac{\mathbf{H}(\theta_c,\rho)}{\mathbf{h}(\theta_c,\rho)}-\frac{\mathbf{F}(\theta_c)}{\mathbf{f}(\theta_c)}+\alpha\frac{\mathbf{F}(\theta_c)}{\mathbf{f}(\theta_c)}\right]$$
$$-b(1-\alpha)\int_{\theta_0}^{\theta_c(\rho)}\frac{\partial}{\partial\rho}\left[\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)}\right]\left[(1-\alpha)\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)}-\frac{\mathbf{F}(\xi)}{\mathbf{f}(\xi)}+\alpha\frac{\mathbf{F}(\xi)}{\mathbf{f}(\xi)}\right]\mathbf{f}(\xi)d\xi \tag{54}$$

$$\frac{\partial E[\mathbf{W}]}{\partial\rho}=\sqrt{2k_0b}\,\mathbf{f}(\theta_c(\rho))(1-\alpha)\frac{\partial\theta_c(\rho)}{\partial\rho}\left[\frac{\mathbf{H}(\theta_c,\rho)}{\mathbf{h}(\theta_c,\rho)}-\frac{\mathbf{F}(\theta_c)}{\mathbf{f}(\theta_c)}\right]$$
$$-b(1-\alpha)^2\int_{\theta_0}^{\theta_c(\rho)}\frac{\partial}{\partial\rho}\left[\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)}\right]\left[\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)}-\frac{\mathbf{F}(\xi)}{\mathbf{f}(\xi)}\right]\mathbf{f}(\xi)d\xi \tag{55}$$

Equation (55) implies that a sufficient condition to determine the sign of $\partial E[\mathbf{W}]/\partial\rho$ is to determine the sign of $\mathbf{H}(\theta,\rho)/\mathbf{h}(\theta,\rho)$ - $\mathbf{F}(\theta)/\mathbf{f}(\theta)$ for all values of θ.

If $\mathbf{H}(\theta,\rho)/\mathbf{h}(\theta,\rho) - \mathbf{F}(\theta)/\mathbf{f}(\theta)\le 0$ for all θ, then $\partial E[\mathbf{W}]/\partial\rho\ge 0$. Then for $\rho>\rho_M$, $\mathbf{H}(\theta,\rho)/\mathbf{h}(\theta,\rho) - \mathbf{F}(\theta)/\mathbf{f}(\theta)\le 0$ ($\partial E[\mathbf{W}]/\partial\rho\ge 0$), for $\rho<\rho_M$, $\mathbf{H}(\theta,\rho)/\mathbf{h}(\theta,\rho) - \mathbf{F}(\theta)/\mathbf{f}(\theta)\ge 0$ ($\partial E[\mathbf{W}]/\partial\rho\le 0$) and for $\rho=\rho_M$, $\mathbf{H}(\theta,\rho)/\mathbf{h}(\theta,\rho) - \mathbf{F}(\theta)/\mathbf{f}(\theta)=0$ thus $\partial E[\mathbf{W}]/\partial\rho=0$ at $\rho=\rho_M$. These three conditions are sufficient and necessary for $\partial E[\mathbf{W}]/\partial\rho$ to have a maximum at $\rho=\rho_M$ for any α.❑

4. Proof of theorem 2.

Using equations (45) and (51), the following expression is obtained for $\partial E[\mathbf{S}+\pi]/\partial\rho$:

$$\frac{\partial E[\mathbf{S}+\pi]}{\partial\rho}=\sqrt{2k_0b}\,\mathbf{f}(\theta_c(\rho))(1-\alpha)\frac{\partial\theta_c(\rho)}{\partial\rho}\cdot\frac{\mathbf{H}(\theta_c,\rho)}{\mathbf{h}(\theta_c,\rho)}$$
$$-b(1-\alpha)^2\int_{\theta_0}^{\theta_c(\rho)}\frac{\partial}{\partial\rho}\left[\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)}\right]\cdot\frac{\mathbf{H}(\xi,\rho)}{\mathbf{h}(\xi,\rho)}\mathbf{f}(\xi)d\xi\le 0 \tag{56}$$

Then theorem 2 follows immediately.❑

5. Proof of theorem 4.

Property (2) of $\mathbf{H}(\theta,\rho)$ and equations (5) and (6) imply that when ρ increases the prices that the government pays also increase, eventually reaching levels that are higher than the monopolist pricing. If ρ is increased even more, the firm will eventually be shut down. This follows from equation (46). Then, it is clear that the expected government gain will tend to zero as ρ increases, because the gain of the government when firm is not allowed to do business is equal to zero.

Also, since by hypothesis $\lim_{\rho\to-\infty}\left(\mathbf{H}(\theta,\rho)/\mathbf{h}(\theta,\rho)\right)=0$ and from equations (5) and (6), the expected government gain tends to the value obtained by using the Loeb-Magat mechanism as ρ decreases.

This condition may be expressed in a more rigorous way. $\lim_{\rho\to-\infty}\left(\mathbf{H}(\theta,\rho)/\mathbf{h}(\theta,\rho)\right)=0$ implies that:

$$\lim_{\rho\to-\infty} z_\alpha(\theta)=\theta \tag{57}$$

$$\lim_{\rho\to-\infty} p(\theta)=\theta \tag{58}$$

$$\lim_{\rho\to-\infty}\left(E[\mathrm{S}(\theta,\rho)]\right)=-k_0+\frac{b}{2}\left(\frac{a}{b}-\theta_1\right)^2 \tag{59}$$

Therefore, if the fixed cost is small, the right side of equation (59) is positive.❑

IV. COMPARISON OF STRATEGIC CHOICES FOR THE REGULATOR.

In the previous chapter it was assumed that the regulator did not know what was the real distribution function for the unknown firm's cost parameter. It will now be assumed that the regulator has somehow determined two alternative distribution functions for the cost parameter, and has to decide which one to use to maximize some measurement of welfare. This scenario is suitable for the use of a strategic payoff matrix. In the following sections this matrix is built for two different measurements of welfare: expected government gain and producer's expected profit.

A. CONDITIONS FOR SELECTING THE DISTRIBUTION FUNCTIONS TO OBTAIN A DOMINANT STRATEGY FOR THE REGULATOR.

Lets assume now that the regulator has two choices for $\mathbf{g}(\cdot)$ (and implicitly $\mathbf{G}(\cdot)$):

$$\mathbf{g}(\theta)=\mathbf{f}_1(\theta), \quad \mathbf{g}(\theta)=\mathbf{f}_2(\theta) \tag{60}$$

because it believes that the company has two possible distributions for Θ.

We can define four values for the expected government gain, depending on the combinations of government belief of $\mathbf{g}(\cdot)$ and the real distribution $\mathbf{f}(\cdot)$, as summarized in the following table:

TABLE 1

Government's belief → / Real distribution ↓	$\mathbf{g}(\theta)=\mathbf{f_1}(\theta)$	$\mathbf{g}(\theta)=\mathbf{f_2}(\theta)$
$\mathbf{f}(\theta)=\mathbf{f_1}(\theta)$	$E_{11}[S(\theta)]$	$E_{12}[S(\theta)]$
$\mathbf{f}(\theta)=\mathbf{f_2}(\theta)$	$E_{21}[S(\theta)]$	$E_{22}[S(\theta)]$

Strategic payoff matrix for the government when there are two different probability functions to choose from.

Table 1 represents the strategic payoff matrix for the government if it chooses to measure welfare in terms of its own expected gain.

It will be a dominant strategy for the government to choose $\mathbf{g}(\theta)=\mathbf{f_2}(\theta)$ if:

$$\left(E_{12}[\mathrm{S}(\theta)] > E_{11}[\mathrm{S}(\theta)]\right) \text{ and } \left(E_{22}[\mathrm{S}(\theta)] > E_{21}[\mathrm{S}(\theta)]\right) \tag{61}$$

because it maximizes its expected gain whatever the real distribution is.

The conditions expressed in (61) can be expanded using equation (23):

$$E_{12}[\mathrm{S}(\theta)] > E_{11}[\mathrm{S}(\theta)] \Rightarrow$$

$$\mathbf{K}(\theta_{c2})\mathbf{F}_1(\theta_{c2}) - \mathbf{K}(\theta_{c1})\mathbf{F}_1(\theta_{c1}) - b(1-\alpha)\left[\int_{\theta_0}^{\theta_{c2}} \Phi_{1,2}(\xi)\mathbf{f}_1(\xi)d\xi - \int_{\theta_0}^{\theta_{c1}} \Phi_{1,1}(\zeta)\mathbf{f}_1(\zeta)d\zeta\right] > 0 \tag{62}$$

$$E_{22}[\mathrm{S}(\theta)] > E_{21}[\mathrm{S}(\theta)] \Rightarrow$$

$$\mathbf{K}(\theta_{c2})\mathbf{F}_2(\theta_{c2}) - \mathbf{K}(\theta_{c1})\mathbf{F}_2(\theta_{c1}) - b(1-\alpha)\left[\int_{\theta_0}^{\theta_{c2}} \Phi_{2,2}(\xi)\mathbf{f}_2(\xi)d\xi - \int_{\theta_0}^{\theta_{c1}} \Phi_{2,1}(\zeta)\mathbf{f}_2(\zeta)d\zeta\right] > 0 \tag{63}$$

Where θ_{c1} and θ_{c2} are defined by (22) using $\mathbf{f_1}(\cdot)$ and $\mathbf{f_2}(\cdot)$ respectively to solve for θ^* and:

$$\Phi_{i,j}(\theta) = \left(\frac{(1-\alpha)}{2}\frac{\mathbf{F}_j(\theta)}{\mathbf{f}_j(\theta)} - \frac{\mathbf{F}_i(\theta)}{\mathbf{f}_i(\theta)}\right)\frac{\mathbf{F}_j(\theta)}{\mathbf{f}_j(\theta)} \tag{64}$$

1. Analysis of the strategic dominance for a simple case.

Lets assume now that a, b, k_0, α, θ_0, θ_1, $\mathbf{f_1}(\cdot)$, $\mathbf{f_2}(\cdot)$, $\mathbf{F_1}(\cdot)$ and $\mathbf{F_2}(\cdot)$ are such that θ_{c1} and θ_{c2} are equal to θ_1, this is, the government is interested in doing business with the contractor in the whole range of values for θ. Then, equations (62) and (63) are reduced to:

$$E_{12}[\mathrm{S}(\theta)] > E_{11}[\mathrm{S}(\theta)] \Rightarrow \int_{\theta_0}^{\theta_1} \left(\Phi_{1,1}(\xi) - \Phi_{1,2}(\xi)\right)\cdot \mathbf{f}_1(\xi)d\xi > 0 \tag{65}$$

$$E_{22}[\mathrm{S}(\theta)] > E_{21}[\mathrm{S}(\theta)] \Rightarrow \int_{\theta_0}^{\theta_1} \left(\Phi_{2,1}(\xi) - \Phi_{2,2}(\xi)\right)\cdot \mathbf{f}_2(\xi)d\xi > 0 \tag{66}$$

Substituting (64) into (65) and (66):

$$-\frac{(1+\alpha)}{2}\int_{\theta_0}^{\theta_1}\left(\frac{\mathbf{F}_1(\xi)}{\mathbf{f}_1(\xi)}-\frac{\mathbf{F}_2(\xi)}{\mathbf{f}_2(\xi)}\right)\left(\frac{\mathbf{F}_1(\xi)}{\mathbf{f}_1(\xi)}-\frac{(1-\alpha)}{(1+\alpha)}\frac{\mathbf{F}_2(\xi)}{\mathbf{f}_2(\xi)}\right)\mathbf{f}_1(\xi)d\xi>0 \qquad (67)$$

and

$$\frac{(1+\alpha)}{2}\int_{\theta_0}^{\theta_1}\left(\frac{\mathbf{F}_1(\xi)}{\mathbf{f}_1(\xi)}-\frac{\mathbf{F}_2(\xi)}{\mathbf{f}_2(\xi)}\right)\left(\frac{(1-\alpha)}{(1+\alpha)}\frac{\mathbf{F}_1(\xi)}{\mathbf{f}_1(\xi)}-\frac{\mathbf{F}_2(\xi)}{\mathbf{f}_2(\xi)}\right)\mathbf{f}_2(\xi)d\xi>0 \qquad (68)$$

If α=0, this is, the regulator does not consider the producer's surplus when applying B-M, (67) and (68) cannot be simultaneously valid, because:

$$\int_{\theta_0}^{\theta_1}\left(\frac{\mathbf{F}_1(\xi)}{\mathbf{f}_1(\xi)}-\frac{\mathbf{F}_2(\xi)}{\mathbf{f}_2(\xi)}\right)^2\mathbf{f}_1(\xi)d\xi<0 \qquad (69)$$

and

$$\int_{\theta_0}^{\theta_1}\left(\frac{\mathbf{F}_1(\xi)}{\mathbf{f}_1(\xi)}-\frac{\mathbf{F}_2(\xi)}{\mathbf{f}_2(\xi)}\right)^2\mathbf{f}_2(\xi)d\xi>0 \qquad (70)$$

Equation (70) is always valid, while equation (69) is never valid. Therefore, if α=0, it is not possible for the regulator to choose a probability distribution function for the B-M mechanism that constitutes a dominant strategy in the sense described above. The regulator must use a different criteria to determine if one probability distribution is "better" than other. This condition follows directly from theorem's 1 corollary.

2. Analysis of the strategic dominance for the general case.

The more general case (Equations (65) and (66)) may be analyzed using the results of the previous chapter. Lets assume that $\mathbf{F_2}(\cdot)$ is more favorable than $\mathbf{F_1}(\cdot)$. Consider first the case where $\mathbf{f}(\cdot)=\mathbf{f_1}(\cdot)$. From equation (51) it is clear that the maximum expected consumer surplus will be obtained for $\mathbf{F_2}(\cdot)$ more favorable than $\mathbf{F_1}(\cdot)$, but if $\mathbf{F_2}(\cdot)$ is much more favorable than $\mathbf{F_1}(\cdot)$, then the expected consumer surplus will ultimately decrease. Therefore, without more conditions it is not possible to determine if $E_{12}[\mathrm{S}(\theta)]>E_{11}[\mathrm{S}(\theta)]$. Lets assume now that $\mathbf{f}(\cdot)=\mathbf{f_2}(\cdot)$. Again, from equation (51) it is clear that the condition expressed in equation (63) will always be valid. Therefore, the strategic dominance of the choice of $\mathbf{f_2}(\cdot)$ over $\mathbf{f_1}(\cdot)$ will depend ultimately on the relative degree of "favorability" of $\mathbf{f_2}(\cdot)$ over $\mathbf{f_1}(\cdot)$, as expressed by equation (62). This result is consistent with the findings of the previous chapter.

Consider now an extension of the simple case. Equation (67) will hold if:

$$\frac{\mathbf{F}_1(\xi)}{\mathbf{f}_1(\xi)} \geq \frac{(1-\alpha)}{(1+\alpha)} \frac{\mathbf{F}_2(\xi)}{\mathbf{f}_2(\xi)} \tag{71}$$

The function $w(\alpha) = (1-\alpha)/(1+\alpha)$ varies between 0 (when α=1) and 1 (when α=0). The simple case described above shows that if α=0, there is no dominant strategy, therefore α will be considered greater than zero. Let $\mathbf{f_1}(\theta)=\mathbf{H}(\theta,\rho_1)$ and $\mathbf{f_2}(\theta)=\mathbf{H}(\theta,\rho_2)$, with $\rho_2>\rho_1$, and $\mathbf{H}(\theta,\rho)$ defined by equation (64) in the previous chapter. Then, the condition expressed by equation (71) will hold if $\rho_2 \leq 1-((1-\alpha)/(1+\alpha))(1-\rho_1)$. This relation limits the increase of "favorability" that the regulator may use, to have a dominant strategy.

If the government uses criterion (1) (from Chapter III) as a basis to select a cost distribution function, this result indicates that there will be never a dominant strategy for the government under these circumstances.

B. IMPACT OF THE STRATEGIC CHOICE IN PRODUCER'S UTILITY.

Table 1 represents the effect of the choice in the expected consumer surplus. It is possible to build a similar table for the producer's expected profit:

TABLE 2

Government's belief → Real distribution ↓	$\mathbf{g}(\theta)=\mathbf{f_1}(\theta)$	$\mathbf{g}(\theta)=\mathbf{f_2}(\theta)$
$\mathbf{f}(\theta)=\mathbf{f_1}(\theta)$	$E_1[\pi_1(\theta)]$	$E_1[\pi_2(\theta)]$
$\mathbf{f}(\theta)=\mathbf{f_2}(\theta)$	$E_2[\pi_1(\theta)]$	$E_2[\pi_2(\theta)]$

Strategic payoff matrix for the producer when the government chooses between two different p.d.f.'s to apply the B-M mechanism.

Where:

$$\pi_g(\theta) = \int_{\theta}^{\theta_{cg}} q_g(\xi) d\xi \tag{72}$$

with $q_g(\xi)$ calculated using **g**(·) and **G**(·), and

$$E_f[\pi_g(\theta)] = \int_{\theta_0}^{\theta_{cg}} \pi_g(\xi) d\mathbf{F}(\xi) = \pi_g(\xi)\cdot \mathbf{F}(\xi)\Big|_{\theta_0}^{\theta_{cg}} - \int_{\theta_0}^{\theta_{cg}} \dot{\pi}_g(\xi)\mathbf{F}(\xi)d\xi = -\int_{\theta_0}^{\theta_{cg}} \dot{\pi}_g(\xi)\mathbf{F}(\xi)d\xi \tag{73}$$

because $\mathbf{F}(\theta_0)=0$ and $\pi_g(\theta_{cg}) = 0$. Equation (72) implies that:

$$\dot{\pi}_g(\theta) = -q_g(\theta) \le 0 \quad (q_g(\theta) \ge 0\ \forall \theta \in [\theta_0, \theta_1]) \tag{74}$$

for all $\theta < \theta_{cg}$.

Lets calculate now $\Delta_1 = E_1[\pi_1(\theta)] - E_1[\pi_2(\theta)]$ and $\Delta_2 = E_2[\pi_1(\theta)] - E_2[\pi_2(\theta)]$.[7]

$$\Delta_1 = -\int_{\theta_0}^{\theta_{c1}} \dot{\pi}_1(\xi)\mathbf{F}_1(\xi)d\xi + \int_{\theta_0}^{\theta_{c2}} \dot{\pi}_2(\xi)\mathbf{F}_1(\xi)d\xi \tag{75}$$

$$\Delta_1 = \int_{\theta_0}^{\theta_{c2}} (\dot{\pi}_2(\xi) - \dot{\pi}_1(\xi))\mathbf{F}_1(\xi)d\xi + \int_{\theta_{c2}}^{\theta_{c1}} -\dot{\pi}_1(\xi)\mathbf{F}_1(\xi)d\xi \tag{76}$$

$$\Delta_1 = \int_{\theta_0}^{\theta_{c2}} -(q_2(\theta) - q_1(\theta))\mathbf{F}_1(\xi)d\xi + \int_{\theta_{c2}}^{\theta_{c1}} -\dot{\pi}_1(\xi)\mathbf{F}_1(\xi)d\xi \tag{77}$$

Under the assumption that $\mathbf{F_2}(\cdot)$ is more favorable than $\mathbf{F_1}(\cdot)$, we know that $\theta_{c1} \ge \theta_{c2}$ and $q_1(\theta) \ge q_2(\theta)$, therefore the first term of equation (77) is positive and because of (46), the second term of the equation is also positive. Thus, $\Delta_1 \ge 0$.

$$\Delta_2 = -\int_{\theta_0}^{\theta_{c1}} \dot{\pi}_1(\xi)\mathbf{F}_2(\xi)d\xi + \int_{\theta_0}^{\theta_{c2}} \dot{\pi}_2(\xi)\mathbf{F}_2(\xi)d\xi \tag{78}$$

$$\Delta_2 = \int_{\theta_0}^{\theta_{c2}} (\dot{\pi}_2(\xi) - \dot{\pi}_1(\xi))\mathbf{F}_2(\xi)d\xi + \int_{\theta_{c2}}^{\theta_{c1}} -\dot{\pi}_1(\xi)\mathbf{F}_2(\xi)d\xi \tag{79}$$

$$\Delta_2 = \int_{\theta_0}^{\theta_{c2}} -(q_2(\theta) - q_1(\theta))\mathbf{F}_2(\xi)d\xi + \int_{\theta_{c2}}^{\theta_{c1}} -\dot{\pi}_1(\xi)\mathbf{F}_2(\xi)d\xi \tag{80}$$

From (80), $\Delta_2 \ge 0$ also. Therefore, the producer is always better off if the government uses a less favorable p.d.f. Even more, if it is assumed that $\mathbf{F_2}(\cdot)$ is strictly more favorable

[7] This analysis is similar to the one conducted in [Ref. 4], pp. 81.

than $\mathbf{F_1}(\cdot)$, then Δ_2-$\Delta_1 \geq 0$ (subtract (77) from (79), and use $\mathbf{F_2}(\cdot) \geq \mathbf{F_1}(\cdot)$). This condition means that the loss for the producer is greater when the producer is more efficient, therefore the more efficient the producer, the more incentive he has to convince the regulator to use a less favorable p.d.f. to apply B-M.

As it was predicted by theorem 3 of Chapter III, the government will always face the resistance of the producer when choosing a dominant strategy. The conditions represented by Table 2 will be valid for all the criteria that the government may use to determine what distribution function it selects for the application of the B-M mechanism.

V. CONCLUSIONS AND RECOMMENDATIONS.

The results obtained in this thesis provide interesting insights about the effects of the choice of a probability density function on the outcome of the application of the Baron-Myerson mechanism to the defense procurement process. First, theorem 1 and its corollary show the importance of having a good knowledge about the firm's privileged information, because it allows the optimization of the government's gain. Second, theorems 2 and 3 elucidate the impact of a change in policy both in total welfare and in the firm's profit, delineating the incentive structure that the Baron-Myerson mechanism creates. Finally, theorem 4 suggests what the effects on the credibility of the policy may be, when there is a high degree of uncertainty about the firm's private information.

The strategic implications of these theorems are very important. Theorem 3 predicts that the producers will be always complaining about their high costs, trying to influence the government so that it reduces the expectations about the efficiency of the processes involved in the manufacture of the goods or services that are being procured. This raises the credibility issue for the contractor. The government should not believe in the information about the probability density functions for the cost parameter that the producer provides, because the firm has clear incentives to misrepresent it. Note that this does not mean that the Baron-Myerson is not successful in making the contractor reveal its true cost. It means that the firm has an incentive to influence the government in its favor, *before* it applies the regulation mechanism.

On the other hand, theorems 1 and 2 create a credibility problem for the government. The contractor will ultimately determine that if the government insists in requiring high efficiency, the outcome for the government may be less desirable than if it reduces the efficiency requirements. This tradeoff between pursuing an aggressive policy (pushing for high efficiency firms) and pursuing a conservative policy (allowing low efficiency firms to operate with acceptable profit levels) may be addressed using theorem 4. If the government has a degree of risk aversion, theorem 4 suggests that it should pursue conservative policies when the uncertainty about the real distribution function is high, because of the impact of requiring high efficiency in the expected government gain.

These conclusions suggest that, if the government believes that it has a good estimate of the probability distribution of the cost parameter, it should use the B-M model with this estimate. On the other hand, if it has doubts about the accuracy of the estimate, it would be better off by pursuing a more conservative approach. When the level of uncertainty is even higher, the government may use a simpler regulation mechanism such as Loeb-Magat.

The limits that separate these three alternatives are not clearly defined. To clarify them, further research should be conducted to determine the behavior of the outcome in the neighborhood of the optimum. Also, new measurements may be developed to assess the degree of uncertainty about the firm's private information. In this sense, theorem 4 is too restrictive to allow a more generalized study of the outcome of the procurement process.

Also, some research could be conducted to determine the dynamic characteristics of the process, specially because a dynamic analysis may improve the estimates of the firm's privileged information[8]. This step is beyond the goals of this thesis.

Summarizing, this thesis has achieved its goals, because it provides an insightful analysis of the effects of the choice of probability distribution on the outcome of the application of the Baron-Myerson mechanism to the defense procurement process. It also provides a starting point for further research in the area.

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[8]From Control Systems theory, the greater the number of observations available from an unknown system, the greater the possibility for determining its behavior and parameters. See for example [Ref. 5], ch.3, where the prediction and observability of a linear system is discussed.

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